General Test

1. (3) How many 2-digit numbers have digits whose sum is a perfect square?
Answer: 17
Solution: *Since the highest sum of any 2 digits is 18, the perfect squares to look for are 1, 4, 9, and 16. 10 has digits that sum to 1. 13, 22, 31, and 40 sum to 4. 18, 27, 36, 45, 54, 63, 72, 81, and 90 sum to 9. 79, 88, and 97 sum to 16. In total, that is 17 numbers.*

2. (3) In the video game BMX-Treme, you earn points for performing stunts and you earn multipliers for scoring combos. There are 4 possible tricks (A,B,C, and D), which earn you 100, 200, 300, and 400 points respectively, give you combo multipliers of x2, x4, x6, and x8 respectively for doing them immediately after another trick, and a final multiplier is given at the end equal to the number of tricks performed, factorial. You can perform a trick more than once, but you can't perform the same trick twice in a row. If you can perform 3 tricks on a single jump, what is the highest number of points you can get in a combo? (As an example, if you perform stunts in the order D, B, C, you will get (400 + 200 + 300)(4)(6)(3!) = 129,600 points.)
Answer: 316,800
Solution: *Since trick D gives the most points and the best multiplier, you need to do it as 1 of your 3 tricks. If you do it as your first trick, you don't get the multiplier but you can do it as your 3rd trick. If you do trick C as your 2nd trick, that will give you a total of (400+300+400)(6)(8)(3!) = 316,800 points.*

3. (4) A sequence of natural numbers follows such that the next number is equal to the number of letters in the previous number. For instance, the sequence starting with 6 goes 6 (six), 3 (three), 5 (five), 4 (four). What is the largest number of terms in such a sequence if the starting number is less than 20?
Answer: 5
Solution: *After quick inspection, it becomes obvious that all sequences end in 4. 5 and 9 have 4 letters in them, but only 17 has 9 letters in it. 3, 7, and 8 have 5 letters in them. 1, 2, 6, and 10 have 3 letters in them. 11 and 12 have 6 letters in them. 13, 14, 18, and 19 have 8 letters in them; 15 has 7 letters in it. The longest sequence would be 12, 6, 3, 5, 4, which has 5 terms.*

4. (4) Joe is thinking of a number which is equal to the sum of the lowest natural numbers that leave a remainder of 4 when divided by 6 and a remainder of 5 when divided by 7. What is Joe's number?
Answer: 122
Solution: *If you subtract 4 from the desired numbers, you get a multiple of 6 that is 1 higher than a multiple of 7. Going through the multiples of 7, you will find a desired number at 40 -> 40 - 4 = 36 , 40 - 5 = 35 . The other desired number is 82 -> 82 - 4 = 78 , 82 - 5 = 77 . 82 + 40 = 122.*

5. (4) Each of the letters M, E, S, and A represents a different even integer between 1 and 9. What is the least possible value of $\left|\frac{M-(E\times S)}{A}\right|$?

Answer: 0

Solution: *To give the least possible value, you want a fraction that is close to 0. If M = E x S , then the least possible value is 0. Since 8 = 2 x 4 , this can be achieved.*

6. (5) What is the sum $\frac{1}{2\times 3}+\frac{1}{3\times 4}+\frac{1}{4\times 5}+\ldots\frac{1}{19\times 20}$? The sum can be expressed as a fraction in the form $\frac{a}{b}$. What is a + b ?

Answer: 29

Solution: *It is handy to recognize that 16=12-13since 1x (x + 1)=1x-1x + 1. Therefore, the sum can be rewritten 12-13+13-14+...119-120. Most of the terms cancel, so you are left with 12-120=920. 9 + 20 = 29.*

7. (5) A deck of 8 cards - 2 aces, 2 kings, 2 queens, and 2 jacks, all diamonds and clubs - are arranged by suit from highest to lowest value, with the ace of clubs being at the top of the deck and the jack of diamonds being at the bottom. The top card is flipped over into a separate pile and separate actions are taken based on what card it is. If the flipped card is an ace or king, the next two cards are placed immediately on the bottom. If the flipped card is a queen or jack, the next card is placed immediately into the pile. Afterwards, another card is flipped and the process repeats. What is the order of cards in the pile, starting from first to last? Express your answer as an 8-digit number with each card representing a distinct digit, as follows: Clubs -> A=1, K=2, Q=3, J=4 ; Diamonds -> A=5, K=6, Q=7, J=8.

Answer: 14,562,837

Solution: *You just need to follow through the process. The first card flipped is the ace of clubs. Since it is an ace, the king and queen of clubs are placed on the bottom of the deck. The next card flipped is the jack of clubs. Since it is a jack, the ace of diamonds is placed in the pile as well. The next card flipped is the king of diamonds. Since it is a king, the queen and jack of diamonds are placed on the bottom. The next card flipped is the king of clubs, so the queens are placed on the bottom. The next card is the jack of diamonds, so the queen of clubs is placed in the pile. Finally, the queen of diamonds is added. This order would give 14562837.*

8. (5) The sum of the last 4 digits of Seth's phone number is 32. How many such 4-digit sequences are there?

Answer: 35

Solution: *The simplest way to approach this is to list out the sets. Since the highest sum of a 4-digit set is 36, there shouldn't be a huge amount of combinations. The sets are: 9995, 9986, 9977, 9887, 8888. For 9995, there are 4! / 3! = 4 permutations. For 9986 and 9887, there are 4! / 2! = 12 permutations for each. For 9977, there are 4! / (2(2!)) = 6 permutations. 8888 has only 1 permutation. Therefore, there are 4 + 12 + 12 + 6 + 1 = 35 sequences.*

9. (6) Each of 10 houses on a street is painted red, green, or blue. Each house is painted only one color and each color is used on at least one house. No two colors are used to paint the same number of houses. In how many ways could the eight houses on the street be painted?

Answer: 29,880

Solution: *This problem requires both combinations and permutations. First list the sets of 3 distinct numbers that sum to 10 -> 127, 136, 145, 235. Each set has 3! = 6 variations. However, you have to factor in the number of combinations for painting the houses based on each variation. For the 127 variations, there are 10! / (9! x 1!) = 10 ways to pick the first house to be painted the color for one house. For the next color, to be painted on two houses, there are 9! / (7! x 2!) = 36 ways to choose. Therefore, for each of the six 127 variations, there are 10 x 36 = 360 combinations. For the 136 variations, there are 10 ways to pick the first house, and 9! / (6! x 3!) = 84 to pick the 3 houses for the next color, so for each of the six 136 variations there are 10 x 84 = 840 combinations. For the 145 variations, there are 10 x [9! / (5! x 4!)] = 10 x 126 = 1260 combinations for each variation. For the 235 variations, there are [10! / (8! x 2!)] x [8! / (5! x 3!)] = 45 x 56 = 2520 combinations for each variation. Therefore, in total there are 6(360 + 840 + 1260 + 2520) = 6(4980) = 29880 combinations.*

10. (6) The Fibonacci sequence is the sequence 1, 1, 2, 3, 5… where each term is the sum of the previous two terms. What is the remainder when the 2015th term of the sequence is divided by 4?

Answer: 2

Solution: *Obviously there must be a pattern for remainders or this question is impossible to answer in the time given. The remainder pattern when dividing by 4 is 1,1,2,3,1,0, which repeats after every 6 terms. To find the remainder of the 2015th term, find 20154= 503 R3. This means that the third entry, 2, is the answer.*